Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-11-10

Conceptual questions

Question 1. First, make sure you understand the formula

$$(*) \qquad \iint_S f(x,y,z)\,dS = \iint_D f(\mathbf{r}(u,v))|\mathbf{r}_u \times \mathbf{r}_v|\,du\,dv$$

(what do all the letters mean?). This is a very powerful and general formula, which generalizes some others you've seen before:

(a) Recall the 2D change of variables formula

$$\iint_R f(x,y)\,dx\,dy = \iint_D f(T(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right|du\,dv$$

from §15.9. Make sure you understand what all the letters mean. Then explain how this formula is a special case of $(*)$.

(b) Recall the surface area formula for graphs $z = f(x,y)$

$$\iint_D \sqrt{(f_x)^2 + (f_y)^2 + 1}\,dx\,dy$$

from §15.5. Make sure you understand what all the letters mean. Then explain how this formula is a special case of $(*)$.

Computations

Problem 1. Parametrize the following surfaces. That is, express x, y, z in terms of your parameters and also specify the relevant region in the parameter plane.

As is always the case with parametrization problems, there are multiple correct answers.

(a) The portion of the cone $x^2 = y^2 + z^2$ that lies between the planes $x = 1$ and $x = 2$.

(b) The part of the sphere $x^2 + y^2 + z^2 = 2$ that lies above the plane $z = -1$.

(c) The parabolic cylinder $y = 1 - z^2$, $y \geq 0$, $-3 \leq x \leq 3$.

(d) The torus obtained by taking the circle of radius 1 centered at the point $(2,0)$ in the xy-plane and rotating it around the y-axis.

(e) The part of the "helical ramp" $z = \arctan(y/x)$ with $y \geq x/\sqrt{3}$, $y \leq x$, and $x^2 + y^2 \leq 4$.

(f) The triangle with vertices $(3,2,3)$, $(4,5,6)$, $(4,6,5)$.